A Simulated Inverted Pendulum to Investigate Human Sensorimotor Control

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***Abstract*—Sensorimotor control regulates balance and stability as well as adaptation to the external environment. We introduce the use of a simulated inverted pendulum to study human sensorimotor control, demonstrating that this system introduces similar control challenges to human subjects as a physical inverted pendulum. Participants exhibited longer stabilization of the system as the pendulum length between the hand and the center of mass increased while the required control input varied in a non-monotonic, yet predictable manner. Finally, we show that the experimental results can be modelled as a PD controller with a time delay of $\tau = 140$ ms, matching the human visuomotor delay. Our results provide evidence of the importance of vision in a control of unstable systems and serve as a proof of concept of a simulated inverted pendulum.**

I. INTRODUCTION

The inverted pendulum is a classic problem in control theory, often used as an assessment tool to test control strategies. This system is both unstable without control and contains nonlinear dynamics. The inverted pendulum is normally implemented with a pivot point mounted on a cart that moves horizontally under the control of a servo motor system. Here we implement this classic model within our virtual reality robotic system in order use this unstable system as a tool to assess human motor control and behaviour.

The use of the inverted pendulum system is also a classic approach to investigating human motor control [1]-[3]. Normally the subject is asked to control a fully mechanical inverted pendulum after training in order to investigate the delays and processes that govern this balancing control. In certain cases the inverted pendulum has been simulated in order to briefly suppress visual feedback and examine the predictive control strategies [1]. Here we fully simulate a virtual inverted pendulum in the horizontal plane in order to further investigate the processes of control. As the system is fully simulated, this allows the experimenter to control every aspect of the feedback (visual, haptic or temporal) in order to understand how each parameter is used by the sensorimotor control system.

Stability is an essential component of human motor control and learning [4]. However the mechanisms used by the human sensorimotor system vary depending on both the task and the effectors used [5], [6]. While reaching movements in an unstable environment have promoted the use of predictive co-contraction to modify the endpoint stiffness of the limb [7]-[9], balance of postural sway has strongly supported the use of feedback control for stabilization [10]-[12]. These differences have been suggested to arise due to the different timescales of the system [5], allowing for the use of delayed feedback for long lengths such as full body sway. The simulation of a virtual inverted pendulum allows for changes in the lengths of the pendulum to be investigated. The objective of this task is to actively balance the inverted pendulum by applying a force to the cart. Here we applied this one degree of freedom pendulum onto a two-dimensional virtual reality robotic system. This apparatus can apply visual and haptic feedback to the participants as they interact with the virtual cart and pendulum. Such a system allows manipulation of the supplied feedback in order to investigate the control strategies of the human subjects in this complex task.

II. MATERIALS AND METHODS

A. Subjects

Six neurologically healthy, right-handed [13] human subjects (1 female) took part in the experiment (mean age 29.0 years). All subjects were naïve to the study purpose and provided written informed consent before participation. The study was approved by the institutional ethics committee at the Technical University of Munich.

B. Experimental apparatus

Participants performed a balancing task of an inverted pendulum simulated with a robotic manipulandum. Subjects were seated with their right arm resting on an airsled and grasping the endpoint handle of the vBOT robotic interface (Fig 1A). The vBOT is a custom made planar robotic interface that generates state-dependent forces on the hand at 1kHz [14] (Fig. 1A). A six-axis force transducer (ATI Nano 25; ATI Industrial Automation) measured the end-point forces applied on the robotic handle by the participants, while handle position was calculated from joint-position sensors (58SA; Industrial Encoders Direct). Position and force data were sampled at 1 kHz. Visual feedback was projected via a computer monitor and a mirror system to the plane of the movement in such a way that the direct visual feedback of the hand was prevented.

C. Experimental paradigm

The inverted pendulum was simulated in the x-y plane with the gravity acting in the negative y direction while corrective movements were performed in the x-axis. Mechanically the

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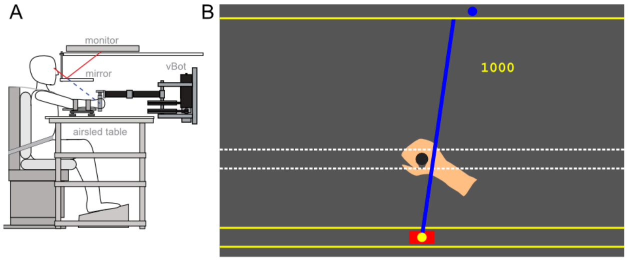

Figure 1. Experimental design. **A**, Subjects attempted to balance an inverted pendulum simulated using a planar robotic manipulandum providing both visual and haptic feedback. **B** A sample screenshot of an experimental trial. The circular cursor at the top of the screen provides visual feedback of the center of mass while the pendulum (blue line) is truncated at the top of the screen. The y-coordinate of the physical hand location (not visible to subjects) is offset with respect to the cart position.

pendulum was represented as a point mass (m = 1 kg) balanced at height (L) above a cart (M = 0.1 kg). The dynamic equations of motion describing the system are:

$$F_x = \ddot{x}(m\sin^2\theta + M) - mL\dot{\theta}^2\sin\theta + mg\sin\theta\cos\theta \quad (1)$$

$$\ddot{\theta} = (g\sin\theta - \ddot{x}\cos\theta)/L \quad (2)$$

where F_x is the lateral force applied by a pendulum on the cart, θ is the angle between the pendulum and the y-axis, x is the position of the cart and g is the gravitational acceleration constant.

The cart, controlled directly by the hand of a subject, was represented as a 1.5 cm by 3.0 cm red block. It was constrained to a single axis of motion in the x direction approximately 30 cm in front of participant's chest by a simulated mechanical channel (stiffness 4000 N/m; damping 2 Ns/m and maximum force value of 25 N). This channel was framed visually on the screen by two yellow lines of 1.0 mm thickness. Any force F_x exerted by the pendulum on the cart was applied on the subject's hand in the x direction. For safety reasons this force was saturated at the absolute value of 5 N and switched off completely when the pendulum angle exceeded 30° from the vertical. In order to maximize visual range, the visual representation of the task was shifted 13.0 cm towards the participant. The x-coordinate of the cart and the handle were always matched. The pendulum itself was represented as a blue line of 3.0 mm thickness connected to the center point of the cart (Fig. 1B). Due to the limitations of the screen size the whole pendulum could not be visualized and therefore it was truncated at the top of the screen. In addition, a blue circle (d = 1.0 cm) moving only in x direction was presented at the top of the screen. This circle represented the lateral position of the center of mass of the pendulum.

Trials were self-paced: subjects initiated each trial by moving the cart to the start position, indicated by a grey rectangle (3.0 cm by 1.5 cm). Participants were notified that they were within the home position by a yellow circle (d = 1.0 cm) appearing at the center of the cart. The trial initiation cue was a short beep followed by the pendulum starting to fall after 600 ms with initial angular velocity $\dot{\theta}$ = 0.01 rad/s. The direction of the fall was randomized with equal probabilities for left and right. Subjects were required to maintain the pendulum in an upright position and with as little oscillation as possible. A trial was considered to have terminated when the angle between the pendulum and the y-axis reached 90° or when the pendulum was successfully balanced for 5.0 s. Subjects were then free to return to the start position and initiate the next trial while the feedback about the previous trial was shown.

In order to provide consistent feedback for participants a score variable (S) was introduced:

$$S = 100\ln\left(\frac{9000}{\sum_{t=0.001}^{5}\theta(t)^2}\right) \quad (3)$$

where t is the time of the sample. If the pendulum was not maintained upright for the duration of the trial, θ = 90° was used for all the remaining samples until the end of the trial.

Participants were introduced to a range of different pendulum dynamics. Specifically, participants were required to control a pendulum of mass m = 1 kg and lengths L = [0.25 m, 0.5 m, 0.75 m, 1 m, 1.5 m, 2 m, 4 m, 6 m, 8 m]. Each experimental block consisted of 20 trials of one given pendulum length. The nine different blocks were presented twice to participants in a pseudo-random order, so that every pendulum length was presented before any condition was repeated. Between blocks a short break was provided (3 s) where an illustration of a teapot was shown to notify participants that conditions would change. This resulted in 40 repetitions of each pendulum condition and a total of 360 trials per participant.

D. *Data analysis*

The data were analysed using Matlab R2016b. Force and kinematic time series were low-pass filtered with a fifth-order, zero-phase-lag Butterworth filter with 40 Hz cutoff. Acceleration data were obtained online by differentiating velocity data and then filtering it with an eight-order Butterworth filter (40 Hz cutoff).

III. RESULTS

A. *Experimental data*

Participants balanced a simulated inverted pendulum of unknown length while being provided the visual feedback of the end point, which coincided with the centre of mass of the pendulum. Participants' ability to maintain the pendulum upright increased with the length of the pendulum until the critical length, where consistent balance was achieved. Beyond this point increasing the length of the pendulum did not improve the stability (Fig. 2).

A small, but consistent, effect across participants was a decrease in the score for the longest length of the pendulum (Fig. 2A). However, a decrease was neither found for the balance time (Fig. 2B) nor angular velocity, which is a measure for the total system instability. Instead, a small increase in corrective movements was recorded for longer lengths (Fig. 2C). Such behaviour may arise due to the higher innate stability of longer pendulums. A long pendulum can be maintained upright even with only a small

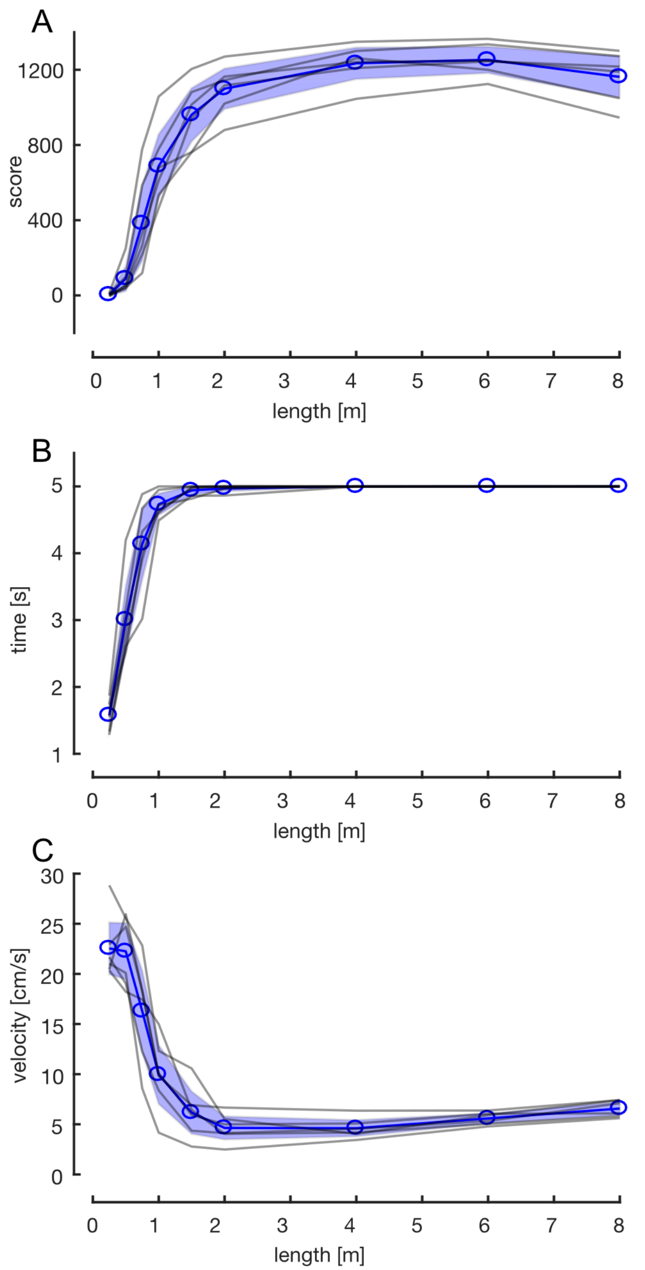

Figure 2. Effect of pendulum length on the controllability of the inverted pendulum. **A**, The score. Individual subject data is represented by grey lines, average response for all subjects is represented by the blue line. Shaded areas represent 95% confidence intervals of the mean. **B**, Average time the pendulum was maintained upright (maximum trial length 5s). **C**, Average velocity of the handle (cart). Cart velocity primarily reflects the subjects' control actions.

angular displacement, resulting in cart movement and increasing the average corrective velocity (Fig. 2C).

B. Computational model

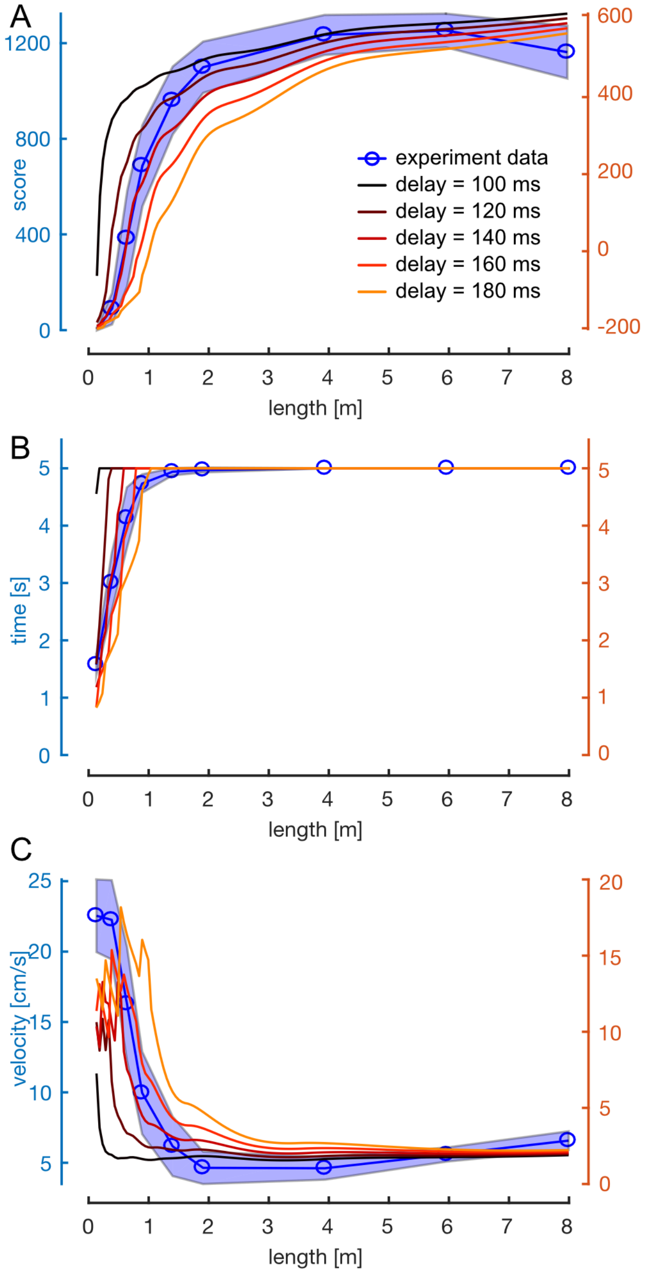

Figure 3. Comparison of experimental data with a delayed PD controller. **A-C**, Score, time balanced, and average cart velocity respectively. Individual lines represent the best fit PD controller (k_p=23, k_d=1.3) with different time delays. Secondary axes represent magnitudes of the controller output (non-normalized). Blue line and shaded region are experimental results as in Fig 2.

The experimental data allowed us to compare control strategies of participants to a PD controller. We simulated a PD controlled virtual pendulum with a feedback delay in Matlab over a range of different controller parameters and compared the results with the experimental data. The performance of the controller was evaluated by comparing normalised output of the controller to the subject data in terms of score, time, angular error, and handle velocity. The fit between simulation and experimental results was then obtained by ranking each parameter set by least-squares error (LSE) for all four state variables. We then selected the best-fit PD controller by minimising the combined ranking for each parameter set resulting in parameters of $k_p = 21$, $k_d = 2.3$ and delay $\tau = 0.14$ s.

The effect of delay can be seen (Fig. 3) where the prediction of the PD controllers with the same PD parameters but different delays are shown. The model prediction was found to closely match subject data for short lengths of the pendulum. At these lengths, due to innate instability of the system a constant control action is required making the controller output comparable to subject data. Subtle differences were observed at longer lengths, likely occurring from increased observation noise due to the higher visual motion for the human subjects. In contrast the PD controller has perfect information about the system, so longer lengths with increased stability will exhibit a monotonic increase in score and smaller control actions.

IV. DISCUSSION

A simplified pendulum is a system consisting of a point mass, connected to a frictionless pin via a rigid weightless link. Such system is stable at the minimum energy configuration with a point mass hanging underneath the pin-joint or resting at the support. In its inverted configuration, the system can achieve marginal stability when no noise is present, however for it to be balanced consistently an online control action is required. The inverted pendulum model is therefore interesting from human motor control perspective, as it allows us to investigate the interaction of these two strategies. For example, a human could stay upright by maintaining a wider stance and co-contracting the muscles, therefore increasing the state-space of marginal stability, or engage the active control of the full-body oscillations.

Another reason why an inverted pendulum system is interesting is that its stability can be varied by changing its parameters such as length or mass. As expected our results show that human subjects have more difficulty to maintain the pendulum upright when its length is decreased (Fig. 2A). Such effects are likely caused by the delays in the human sensorimotor system; as a control action is applied to a delayed state of the pendulum, a less stable system may be too far away from the original state at the time when control input reaches the system, to be successfully balanced.

In this paper we present a simulated inverted pendulum system that could be employed to evaluate the human behavior when controlling external dynamics. We show that from a control perspective a simulated pendulum behaves similarly to a real pendulum as it is increasingly easier to maintain with an increasing length. Moreover, in our study the control input by the subjects was minimized at medium lengths (Fig. 2C, L = 2 m, 4 m). We would expect a similar behavior while controlling a real pendulum due to competing mechanisms: a decreasing control input with increasing length due to improving stability, and an increasing control input with increasing length due to the fact that same angular deviation moves center of mass of the pendulum further away for a longer system.

The results of our study can be used to examine the sensory feedback mechanisms used by the subjects. Different input modalities e.g. vision or proprioception, have different delays [15], [16] and different noise characteristics [17]. Therefore, comparison of our results with a time delayed PD controller can be used to estimate the way humans integrate sensory information to control external objects. A controller with time delay τ = 140 ms best explained the experimental data (Fig. 3). Similar delays are known to be present in a human visuomotor system [15], thus suggesting the importance of visual feedback in the control of an inverted pendulum. This may be explained by the fact that an inverted pendulum is largely controlled within a state-space of small angle deviations. At small angles the forces applied on the hand due to the pendulum drifting sideways are negligible and vision is a more reliable estimator of the state. Here we introduce a simulated pendulum for the study of control processes underlying stabilization. Future studies will investigate the relative role and adaptation of visual and haptic feedback to this control.

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