# LQG framework explains performance of balancing inverted pendulum with incongruent visual feedback

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*Abstract*— **Successful manipulation of objects requires forming internal representations of the object dynamics. To do so, the sensorimotor system uses visual feedback of the object movement allowing us to estimate the object state and build the representation. One way to investigate this mechanism is by introducing a discrepancy between the visual feedback about the object's movement and the actual movement. This causes a decline in the ability to accurately control the object, shedding light about possible factors influencing the performance. In this study, we show that an optimal feedback control framework can account for the performance and kinematic characteristics of balancing an inverted pendulum when visual feedback of pendulum tip did not represent the actual pendulum tip. Our model suggests a possible mechanism for the role of visual feedback on forming internal representation of objects' dynamics.**

## I. Introduction

During interaction with the environment we often manipulate external objects. Since we cannot directly measure the state of the object, we rely on visual [1] and proprioceptive feedback [2] to build an internal representation of the object's dynamics that in turn allows us to better control the object [3]. One critical objective while controlling an object is to maintain its stability. This objective usually requires us to move in a specific manner to keep the object stabilized. While we have the capability of actively stabilizing unstable objects, there is still an open question regarding the underlying mechanism that is used to do so.

Different studies have attempted to explain the characteristics of movements while manipulating dynamic objects based on feedforward control [1, 4], or feedback correction [5, 6]. By building computational models, based on different control theories, we have different mechanisms which can account for the way we interact and manipulate objects. However, in some cases, such as balancing an inverted pendulum, there is little difference in the predictions of these models [6].

One method to distinguish between these models is to introduce a discrepancy between the actual dynamics of the object and the sensory feedback of the object's movement. Manipulations, such as introducing delay between sensory feedbacks [7] or distorting the visual feedback [8], change the way we interact with the object, which can reveal possible underlying control mechanisms. Thus, the comparison between model predictions and experimental data allows us to test to what extent each model can capture the behavior under the effect of the manipulation.

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In this study we tested the predictions of a continuous optimal feedback controller balancing an inverted pendulum with incongruent visual feedback. We compare this to a data set of participants balancing an inverted pendulum in a haptic augmented virtual reality system. Using the virtual reality simulation, the visual feedback provided to the participants was manipulated so that participants received feedback from a longer or shorter pendulum length compared to the pendulum length that was used to calculate the actual motion. We show that the LQG framework can capture the decline in balancing performance as well as the velocities participants exhibited during the balancing task.

## II. Material and Methods

### A. Experimental Data

We used a data set of participants balancing a virtual inverted pendulum presented in a previous study [8]. During the experiments, participants sat in front of a virtual reality system while holding the handle of a manipulandum. The participants looked at a mirror showing the projection of an LCD screen placed horizontally above it. The virtual environment simulated an inverted pendulum that participants had to control (Fig. 1A). The pendulum was represented as a point mass ($m$ = 1 kg) balanced at height ($L$) above a cart ($M$ = 0.1 kg). The following dynamical description was used to simulate the pendulum system:

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} \dfrac{-F\cos\theta - mL\dot{\theta}^2 \sin\theta\cos\theta + (M+m)g\sin\theta}{L(M+m\sin^2\theta)} \\ \dfrac{F + m\sin\theta(L\dot{\theta}^2 - g\cos\theta)}{M+m\sin^2\theta} \end{bmatrix} \tag{1}$$

where $F$ is the lateral force applied by a pendulum on the cart, $\theta$ is the angle between the pendulum and the y-axis, x is the position of the cart and $g$ is the gravitational acceleration constant. The length of the pendulum changed between experiments and was longer than the screen size. To overcome this limitation while providing participants with information about the tip position, a blue circle moving only in x direction was presented at the top of the screen. This circle represented the lateral motion of the visual feedback point of the pendulum, which depended on the experimental condition (Fig. 1B). For complete description of the experimental setup, please see [8].

The data set included two experimental conditions. In experiment 1, the pendulum length had 9 possible values

($L = 0.25,\ 0.5,\ 0.75,\ 1,\ 1.5,\ 2,\ 4,\ 6,\ 8\ [m]$) and the visual feedback about the pendulum tip was consistent with the actual tip position. In experiment 2, the pendulum length was constant ($L = 2\ [m]$) however the visual feedback about the tip position was calculated using different pendulum lengths (Fig. 1C):

$$x_{tip} = x + L_{\mathrm{mod}} \sin(\theta) \qquad (2)$$

$L_{\mathrm{mod}}$ value was picked out of 9 possible values ($L_{\mathrm{mod}} = 0.25,\ 0.5,\ 0.75,\ 1,\ 1.5,\ 2,\ 4,\ 6,\ 8\ [m]$). This creates a discrepancy between the actual tip position relative to the cart position. At the beginning of each trial the pendulum initial angular velocity was set to $\theta_0 = \pm 0.01\ [rad/\mathrm{sec}]$.

Participants were asked to keep the pendulum in an upright position and with as little oscillation as possible for 5 seconds by controlling the cart position. The balancing performance was evaluated after each trial and the score was displayed on the screen. The equation for calculating the score was:

$$S = 100 \cdot \ln\left(\frac{9000}{\sum_{t=0.001}^{5} \theta(t)^2}\right) \qquad (3)$$

where $t$ is the time of the sample. If the pendulum was not maintained upright for the duration of the trial, θ=90º was used for all the remaining samples until the end of the trial.

### B. Optimal Feedback Control Framework

We used an optimal control model based on [6] which is shown in Fig. 2. The system block represents the non-linear dynamics equation of the pendulum. We added Gaussian noise to the dynamics equations (1).

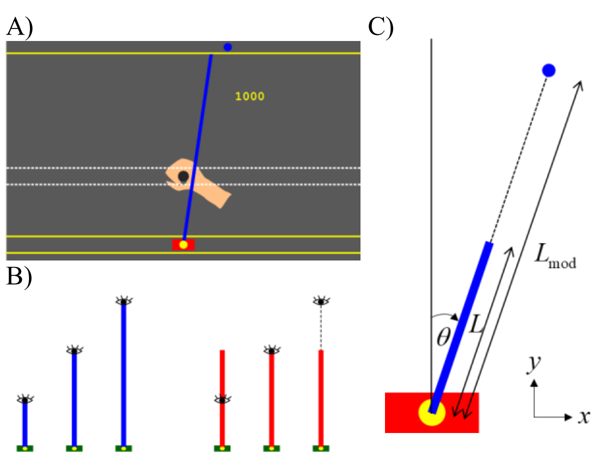


Figure 1. Experimental setup. **A**. The display of the virtual inverted pendulum. The cart (red square) position was set according to participant's hand position. The angle of the pendulum (blue line) and the horizontal position of the pendulum tip (blue circle) were calculated according to equations (1). **B.** Experimental conditions. In experiment 1, the visual feedback about the pendulum tip was consistent with the actual pendulum tip position. In experiment 2, the pendulum length was constant while the visual feedback given about the pendulum tip was calculated according to shorter, longer or similar pendulum length. **C**. Schematic representation of the pendulum with incongruent visual feedback about tip position. In this experimental condition, the actual length of the pendulum was set to $L$ while the visual feedback was calculated according to $L_{\mathrm{mod}}$.

The output of the system, subjected to measurement noise, is fed into an observer, implemented using a Kalman filter, which estimates the state variables $\theta, \dot{\theta}, x, \dot{x}$. The estimated state variables are then used to calculate a control signal using full state controller that was derived by solving an LQR problem. For the observer and state feedback controller we used a linearized representation of the system (1):

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{mg}{M} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{(M+m)g}{L_{\mathrm{mod}} M} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L_{\mathrm{mod}} M} \\ 0 \\ \frac{1}{M} \end{bmatrix} F \qquad (4)$$

We assume that the incongruent visual information changes the estimated pendulum length in such a way that it matches the visual feedback and not the actual mechanical length in the case where the two are not consistent. The observer input, $y_{in}$, consists of two parts, the cart position and the horizontal distance between the cart and the pendulum tip position, subjected to additive noise, $v$:

$$y_{in} = \begin{bmatrix} x \\ x + L_{\mathrm{mod}} \theta \end{bmatrix} + v \qquad (5)$$

We assume that participants used the visual feedback provided about the tip position in order to estimate the pendulum length, $L_{\mathrm{mod}}$. Thus, for experiment 1 the estimated length matched the actual length of the pendulum, however, for experiment 2, the estimated length could be shorter or longer than the actual pendulum length. This assumption suggests that both the observer and feedback controller will change due to the estimated pendulum length.

We introduced a delay of 100 ms between the output of the observer and the feedback controller. This value

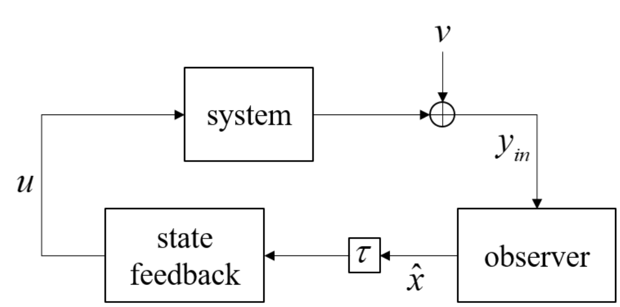


Figure 2. Optimal feedback control block diagram. We implemented the system block using (1). The output of the system block is a measurements vector which appears in Eq. 5 subject to measurement noise $v$. The observer block uses the measurement vector $y$ in order to estimate the state variables of the pendulum, $\hat{x}$. The estimated state variables, are then delayed by $\tau$ ms and used by the state feedback control in order to calculate the optimal control signal u.

approximates the response time to visually induced perturbations [9, 10].

*C. Simulations*

To simulate the pendulum dynamics, we used the same values of parameters ($L, M, m, \dot{\theta}_0$) as in the experiments. We changed the value of pendulum length $L$, according to the values used in experiments 1 and 2. In addition, we simulated the visual feedback that was available to the observer according to the values of $L_{mod}$ that were used in both experiments. Thus, for the simulations of experiment 1 we had 9 scenarios in which the values of $L$ and $L_{mod}$ were equal. For the simulations of experiment 2 we again had 9 scenarios with $L$=2 and changing $L_{mod}$. For each scenario we simulated 10 trials, 5 seconds each, of pendulum balancing by the controller.

For all simulations we used similar $Q$ and $R$ matrices to find the optimal full state feedback gains:

$$Q = \begin{bmatrix} 100 & & & \\ & 10 & & \\ & & 0.1 & \\ & & & 1 \end{bmatrix}, \quad R = 0.1$$

For each movement we calculated the score in a similar way to the score given to the participants. In addition, we extracted two movement metrics from the experimental data and the simulations of the movement. We examined the average time the pendulum was maintained in an upright position, and the average absolute transient velocity of the cart. For the transient velocity response, we took the velocities generated during first second of each trial.

## III. Results

In this study, we show that an optimal feedback control scheme, which balances an inverted pendulum under the condition of incongruent visual feedback, can predict the performance and movement characteristics of participants balancing an inverted pendulum under the same conditions. Examples for the pendulum angle as a function of time is depicted in Figure 3.

Overall, we compared the task performance between the LQG model and participants in terms of the score (3). We observed that the score the model received during the balancing task had similar trend to the one exhibited by the participants (Fig. 4A and 4B). Under the condition of experiment 1, the model's score increased as a function of pendulum length until $L$=1.5 m and saturated for longer lengths. For experiment 2, the model's score exhibited a concave shape with a maximum value at pendulum length equal to 2 m which was the only length where the visual length matched the actual length. We observed similar trends in the performance of the participants.

The LQG model could also predict the amount of time that participants could keep the pendulum in an upright position (Fig. 4C and 4D). For experiment 1, for short pendulum lengths (0.5-1 m) the model could not keep the pendulum stable for the entire duration of the trial (<5 sec). For experiment 2, the duration of stabilizing the pendulum

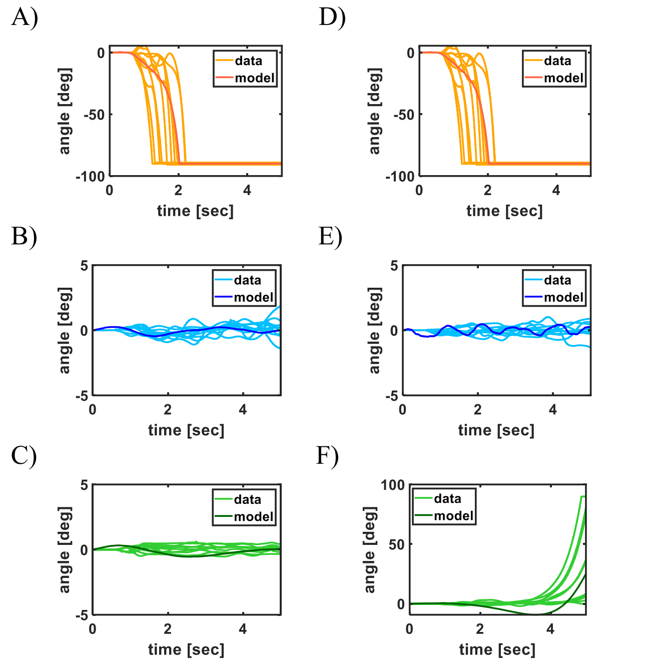


Figure 3. Examples of pendulum angle as a function of time. Examples from experiment 1 is presented in A, B, and C for conditions L=0.25, 1.5, and 8 respectively. In each figure we plotted 10 representative trials performed by one subject using light orange, blue and green lines. Dark orange, blue and green line represent the LQG model predictions for each condition. Examples from experiment 2 using the same notation are presented in D, E, and F.

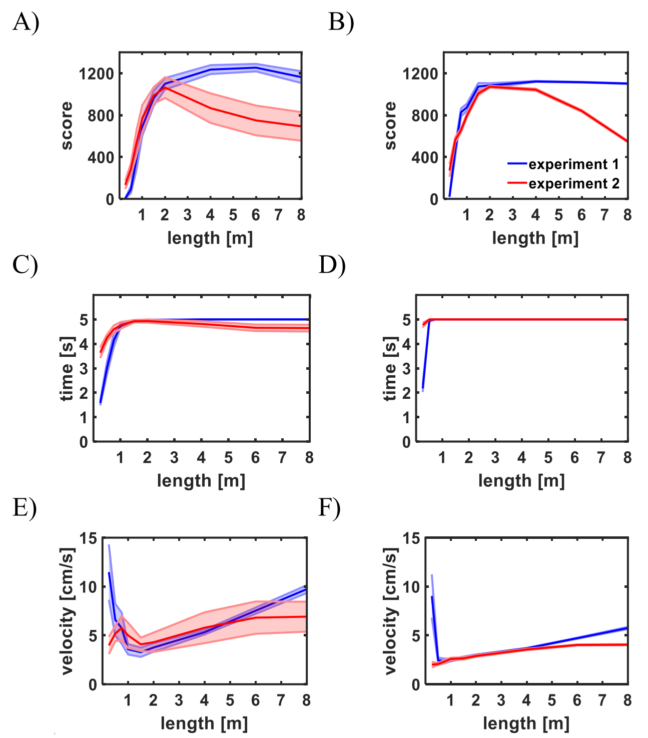


Figure 4. Comparison of task performance and movement kinematics between experimental result and model prediction. A, C and E represent the average score, time that the pendulum maintained in upright position and cart transient velocity respectively as a function of the visual pendulum length. Blue lines represent the mean value of experiment 1 data where the mechanical length and visual length were similar. Red lines represent the mean value of experiment 2 data where the visual length of the pendulum could be smaller, larger or match the actual length. Panels B, D, and F represent the same metrics as in A, C, and E but for the simulation results. Shaded area represents the standard error.

had similar values across visual lengths since the actual pendulum length was set on 2 m. These trends were also evident in the experimental data.

Finally, we observed similar agreement between model's prediction and experimental results for the transient velocity values. For experiment 1, both the model and participants exhibit a convex shape of the velocity as a function of pendulum length. For experiment 2, the velocities remained similar as a function of the visual pendulum length with a slight increase during balancing of the pendulum with longer visual lengths.

## IV. Discussion

We show here that a controller based on continuous optimal feedback control can predict the performance of participants balancing an inverted pendulum. Our model captures the performance of participants during balancing an inverted pendulum with different pendulum lengths as well as when the visual feedback about the pendulum tip represents different pendulum lengths compared with the actual mechanical length. We built the LQG controller which uses visual length as opposed to mechanical length in order to generate the control input. In the case of incongruent visual feedback, this estimation is inconsistent with the actual pendulum length. The simulation of this controller matches well with the experimental data suggesting similar processes in the human motor control system.

Other control theories may also be adequate to predict the behavioral performance [11]. Models based on intermittent optimal feedback control were shown to produce similar predictions to models based on continuous optimal feedback control for pendulum stabilization task [6, 12]. In addition, models based on PD control can also capture performance of participants balancing an inverted pendulum [13]. While we show here that a continuous optimal feedback controller can explain simple pendulum stabilization under normal conditions and with altered visual feedback, it remains to test what are the predictions of other models, such as [14], and whether such visual manipulation could distinguish between the models.

While the LQG model could predict the performance of participants under the different conditions, there are still some differences between the predictions and experimental results. Possible factors that could contribute to this difference may be the effect of sensory delay, measurement noise or the linearization of the pendulum system. Both the delay and noise factors decrease the ability of the feedback controller to accurately control system. While the observer is capable of reducing the effect of measurement noise, we did not include any component that could reduce the effect of delay in the model. Such elements may include a predictor [6], which provide an estimation of future state and so reduces the effect of delay. The effect of delay can also be compensated by the state feedback controller. Using simulation where we changed the delay values and are not presented here, we observed similar task performance trends with increasing decrease in performance as delay increased. Quantifying the compensation of the feedback gains to delay is left as future work.

## V. Conclusion

We studied how incongruent visual feedback may influence the predictions of an optimal feedback controller in an inverted pendulum balancing task. The inconsistency between the internal representation of the pendulum dynamics and the actual dynamics caused a decline in the balancing performance of the controller due to inaccurate estimation of the state variables. The performance trend of the model matched the performance trend of participants as seen in the experimental data. Thus, we suggest that participants relied on the visual information in order to build an internal representation of the pendulum dynamics. Understanding the effect of visual information in tasks which require feedback based corrections is important to a wide range of applications, including rehabilitation or human-machine interfaces.

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